Assignment # 2	Due on October 30th in class

1. In a metric space (X, d), prove that any open ball is an open set, and any closed ball is a closed set.

2. In a metric space (X, d), for any $M \subset X$, prove that $\mathrm{Int}(M)$ is an open set.

Hint: For any $x \in \mathrm{Int}(M)$, from the definition of interiors, there exists $\varepsilon > 0$ such that $B(x; \varepsilon) \subset M$. Based on this, prove the following first: For any $y \in B(x; \varepsilon/3)$, $B(y; \varepsilon/3) \subset B(x; \varepsilon)$.

3. In a metric space (X, d), use $\mathcal{T}$ to denote the collection of all the open sets. Prove that we have the following properties for $\mathcal{T}$:

i) $\varnothing \in \mathcal{T}$ and $X \in \mathcal{T}$.

ii) Let $\mathscr{A}$ be an index set, and assume $S_i \in \mathcal{T}$ for all $i \in \mathscr{A}$. Then $\bigcup_{i \in \mathscr{A}} S_i \in \mathcal{T}$.

iii) Let $K_1, \cdots, K_n$ be in $\mathcal{T}$. Then $\bigcap_{i=1}^{n} K_i \in \mathcal{T}$.

Remark: These three properties above give an abstract characterization of "open sets" in topological spaces.